Exercise 40

Brain weight B as a function of body weight W in fish has been modeled by the power function $B = 0.007W^{2/3}$, where B and W are measured in grams. A model for body weight as a function of body length L (measured in centimeters) is $W = 0.12L^{2.53}$. If, over 10 million years, the average length of a certain species of fish evolved from 15 cm to 20 cm at a constant rate, how fast was this species' brain growing when the average length was 18 cm?

Solution

Write B, the brain mass, in terms of L, the body length.

$$\begin{aligned} B &= 0.007W^{2/3} \\ &= 0.007(0.12L^{2.53})^{2/3} \\ &= 0.007(0.12)^{2/3}L^{5.06/3} \end{aligned}$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\begin{aligned} \frac{d}{dt}(B) &= \frac{d}{dt}\left[0.007(0.12)^{2/3}L^{5.06/3}\right] \\ \frac{dB}{dt} &= 0.007(0.12)^{2/3}\left(\frac{5.06}{3}\right)L^{(5.06/3)-1}\cdot\frac{dL}{dt} \\ &\approx 0.007(0.12)^{2/3}\left(\frac{5.06}{3}\right)L^{(5.06/3)-1}\cdot\frac{20-15}{10{,}000{,}000} \end{aligned}$$

Therefore, when the average length was 18 cm, the rate of change of the brain mass was

$$\left.\frac{dB}{dt}\right|_{L=18} = 0.007(0.12)^{2/3}\left(\frac{5.06}{3}\right)(18)^{(5.06/3)-1}\cdot\frac{20-15}{10{,}000{,}000} \approx 1.04513\times10^{-8}\ \frac{\text{grams}}{\text{year}}.$$